Change of the nature of the multicritical point in magnetically induced splay-twist Fréedericksz transitions

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The phase diagram in the vicinity of the Fréedericksz transition from the uniform to the deformed homogeneous or periodic states of a nematic slab under a magnetic field with rigid boundary conditions was qualitatively calculated in the mean-field approximation for intermediate geometries between the splay and twist Fréedericksz transition geometries. It was found that the multicritical point where the uniform state, the homogeneous deformed, and the periodic deformed states meet is of the Lifshitz type only for a limited range of the Frank elastic constants ratio K_2/K_1. [S1063-651X(98)09507-5]

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Lonberg and Meyer [1] were the first to report the Freedericksz transition in the splay geometry where the usual homogeneous splay state is replaced by a periodic distorted splay-twist state for a suitable range of the K_2/K_1 elastic constants ratio. Since then, several authors have studied this subject, extending the analysis to the other magnetic field-director geometries, considering the simultaneous presence of a magnetic and electric field and considering also the effect of soft boundary conditions [2–23]. Intermediate geometries were also studied by Kini [2].

In this work we have investigated the phase diagrams near the Fréedericksz transition from the uniform to the deformed homogeneous or periodic states of a nematic with a positive diamagnetic susceptibility anisotropy, contained in between two parallel plates under a magnetic field with rigid planar boundary conditions for intermediate magnetic field-director geometries in between the splay and twist Fréedericksz transition geometries.

The calculated phase diagrams show the existence of three different nematic director structures, separated by transition lines of first or second order that meet at a multicritical point as found in the pure splay and twist geometries [11–13,15]. The control parameters in the phase diagrams obtained are the magnetic field strength and either the elastic constant ratio K_2/K_1 or the magnetic field orientation.

To determine the uniform to either periodic or homogeneous distorted states transition lines, we have performed a linear stability analysis of the uniform state. We calculated the Frank elastic free energy plus the magnetic field contribution on the unit of volume, keeping terms up to fourth order in the order parameters, which are the coefficients of the Fourier series expansion considered for the angles that parametrize the nematic director. The limits of the stability region for the uniform state were then calculated. To determine the coexistence line, the director field in the distorted states was obtained by minimization of the total free energy per unit volume and the free energy minima corresponding to the homogeneous and periodic distorted states compared yielding the Maxwell set.

The magnetic field-director geometry studied is shown in Fig. 1. The pure splay and twist cases correspond to $\psi=0$ and $\psi=\pi/2$, respectively.

The magnetic field is given by

$$H_x=0,\quad H_y=H\sin(\psi),\quad H_z=H\cos(\psi). \tag{1}$$

The director considered is

$$n_x=\cos(\theta)\cos(\phi),\quad n_y=\cos(\theta)\sin(\phi),\quad n_z=\sin(\theta). \tag{2}$$

The *Ansatz* considered is

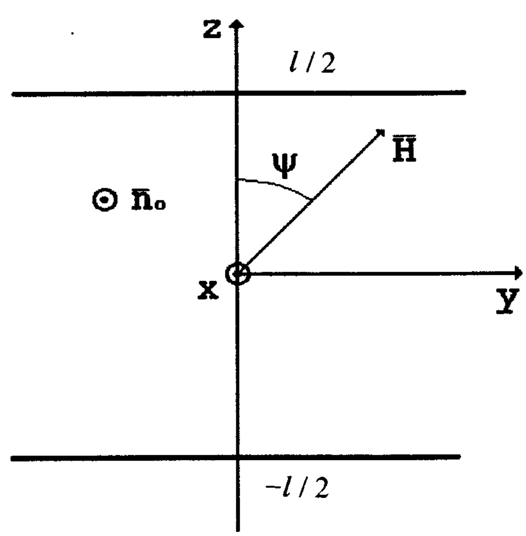

FIG. 1. Magnetic field-nematic director Fréedericksz transition geometry analyzed in this study. The plates are at $z=\pm l/2$.

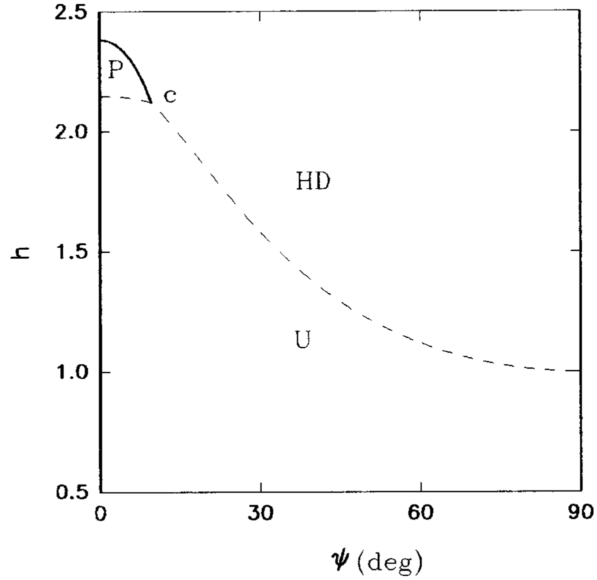

FIG. 2. Phase diagram for the splay-twist mixed geometry for $r=0.2$. The dashed line represents continuous transitions and the full line is a coexistence line obtained with $K_3=K_1$. U, uniform state; P, periodic distorted state; HD, homogeneous distorted state; c, multicritical point.

$$\theta=[\theta_{00}+\theta_{01}\cos(q_y y/l)]\cos(\pi z/l)+\theta_{11}\sin(q_y y/l)\sin(2\pi z/l), \tag{3a}$$

$$\phi=\phi_{11}\sin(q_y y/l)\sin(2\pi z/l)+[\phi_{00}+\phi_{01}\cos(q_y y/l)]\cos(\pi z/l). \tag{3b}$$

The *Ansatz* is appropriate to describe the director for small deformations by including the lowest-order Fourier series terms for the y and z dependence of the parametrizing angles of the director compatible with the z boundary conditions and allowing for the existence of a periodic distortion in the y direction. The possibility of periodic distortions in the x direction and in arbitrary directions in the x,y plane were also investigated but were not favored. The order parameters are θ_{00}, θ_{01}, θ_{11}, ϕ_{00}, ϕ_{01}, and ϕ_{11}. The value of q_y is also determined in the minimization process. The Frank free energy plus the magnetic contribution per unit of volume is [24]

$$F=\frac{1}{l\lambda_y}\int_{\lambda_y}dy\int_{-l/2}^{l/2}dz\,\frac{1}{2}\{K_1(\nabla\cdot\vec{n})^2+K_2(\vec{n}\cdot\nabla\times\vec{n})^2+K_3(\vec{n}\times\nabla\times\vec{n})^2-\chi_a(n\cdot H)^2\}. \tag{4}$$

The reduced magnetic field considered is

$$h=\frac{l}{\pi}\left(\frac{\chi_a}{K_2}\right)^{1/2}H. \tag{5}$$

In Fig. 2 is presented a typical phase diagram obtained for $r\equiv K_2/K_1=0.2$. The value of h at the multicritical point (h_c) as a function of r and ψ_c is given by

$$h_c=\left(\frac{1}{\cos^2(\psi_c)(r-1)+1}\right)^{1/2} \tag{6}$$

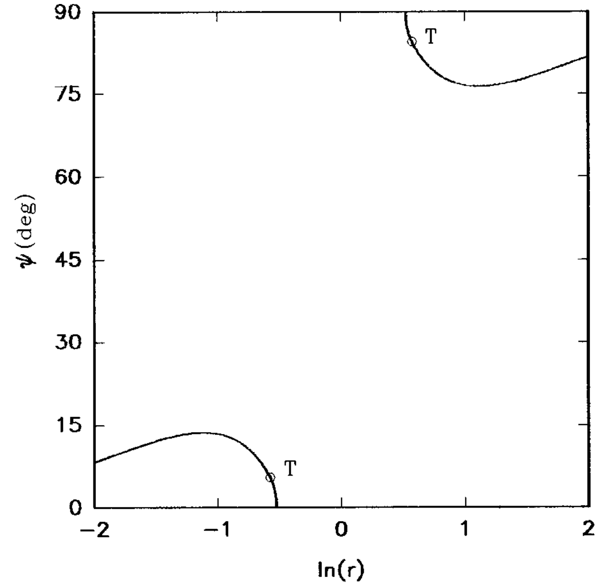

FIG. 3. Multicritical point locus in the ψ versus r plane for the splay-twist mixed geometry. T denotes the point where the nature of the multicritical point changes.

in agreement with the result of Deuling [25]. For ψ_c no simple analytical expression can be given, it is found as the value of ψ for which the threshold fields for the onset of the periodic and aperiodic distorted states are equal. Its dependence on r is plotted in Fig. 3 where the two branches of the (r,ψ_c) curve are isomorphic for the transformation $(r,\psi_c)\leftrightarrow(1/r,\pi/2-\psi_c)$ as first noticed by Kini [2]. Figure 4 shows an enlargement of Fig. 3 for the $r<0.3$ region, where we will concentrate our study.

The behavior of the multicritical point can be understood from Fig. 5, where the selected value of q_y (q_{ys}), found by minimizing the transition magnetic field, is plotted as a function of ψ and r. For r in between r_T and $r_c\approx0.298$ the wave vector q_{ys} over the transition line uniform-periodic distorted goes to zero continuously at the multicritical point making it

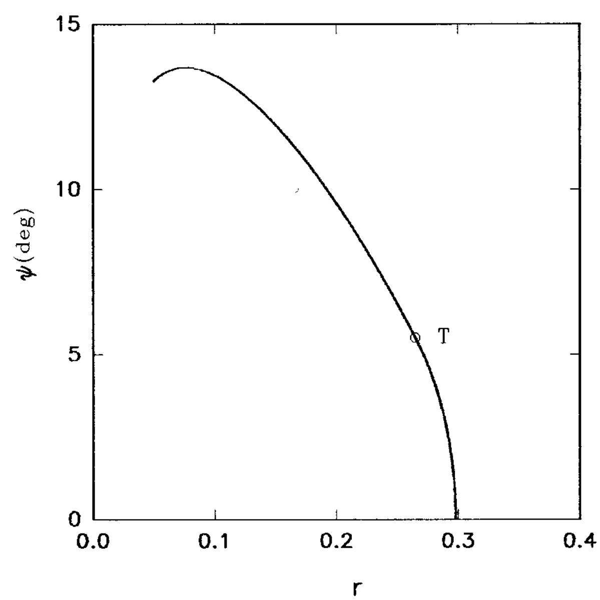

FIG. 4. Enlargement of Fig. 3 for $r<0.3$.

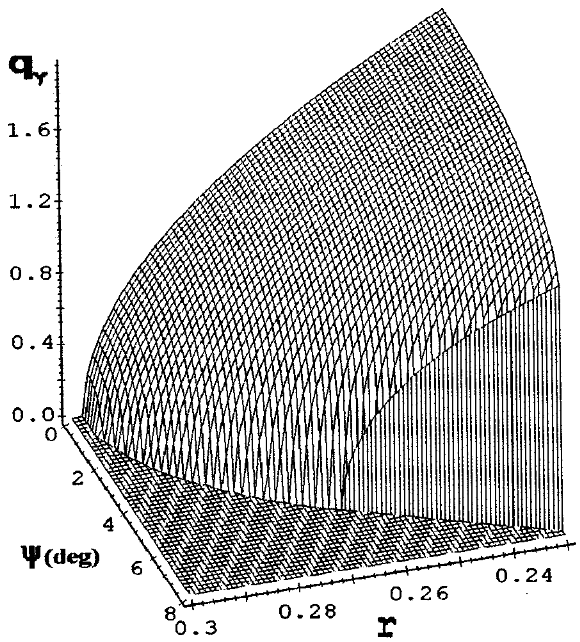

FIG. 5. Selected wave vector q_{ys} as a function of ψ and r.

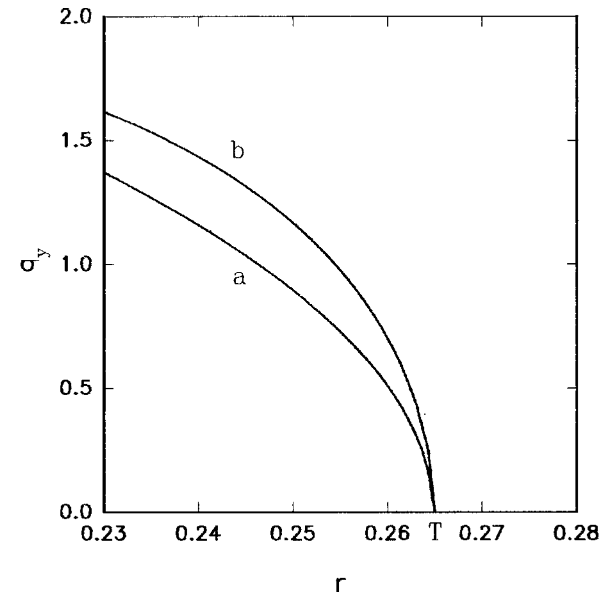

FIG. 6. Jump in q_{ys} along the line of first-order transitions in q_y as found numerically (a) and given by the tricritical model analogy (b).

a Lifshitz point [11–13,26]. For $r<r_T$ the multicritical point is no longer a Lifshitz point and becomes a bicritical point. Below the point T the wave vector q_{ys} changes abruptly when crossing the line of critical points. The intersection of the surface $q_{ys}(\psi,r)$ with the plane $q_{ys}=0$ gives the line shown in Fig. 3. The coordinates and the nature of the point T can also be obtained using a mean-field phase transition analogy, where q_y now plays the role of an order parameter and the role of a potential is played by the reduced magnetic field squared $h_r^2(q_y)$. We first write $h_r^2(q_y)$ in the form of a power series in q_y up to the sixth order:

$$h_r^2(q_y)\equiv\frac{h_p^2(q_y)-h_o^2}{h_o^2}=aq_y^2+bq_y^4+cq_y^6, \tag{7}$$

where a, b, and c are long functions of ψ and r, and $h_p(q_y)$ and h_o are the threshold fields for the onset of the periodic and aperiodic distorted states. Since $c>0$ for the interesting range of the control parameters, this expression is of the form of a classical (mean-field) tricritical model [27]. The point T is then simply the tricritical point and its coordinates in the (ψ,r) plane can then be obtained from the resolution of the system:

$$a=0, \tag{8a}$$

$$b=0, \tag{8b}$$

yielding $[\psi_T=5.504\,98...\ (°),\ r_T=0.264\,77...]$ which is the exact result obtainable from the solution of the system of equations $\partial h^2(q_y)/\partial(q_y^2)=0$, $\partial^2 h^2(q_y)/\partial(q_y^2)^2=0$. In the light of this analogy, the surface shown in Fig. 5 is just the analog of an equation of state in a classical tricritical model of phase transitions.

As in the tricritical model, we found that the jump in q_{ys} from zero to a finite value along the line of first-order transitions decreases continuously to reach the zero value at the tricritical point T. The numerical value of the jump in $q_{ys}(\psi,r)$ along that line shows that it is proportional to $(r^*)^{1/2}\equiv(r-r_T)^{1/2}$. The jump in $q_{ys}(\psi,r)$ along the line of first-order transitions as found numerically is compared with the tricritical model value of $(-4a/b)^{1/2}$ along the same line in Fig. 6.

The method that we employed for the study of the phase diagrams relies on the small value of the amplitude of the distortions considered. It is then only valid for magnetic fields not much stronger than the values for the transition uniform-distorted, making its use a poor approximation for studying the higher magnetic field range. When flexoelectricity is also considered, an extra term appears in expression (4) [28], but this term will not contribute to the critical magnetic field. The only effect that flexoelectricity will have on the work presented is to alter somewhat the position of the coexistence line shown in Fig. 2, but not the point where it meets the continuous transition lines.

The phase diagrams previously described were obtained with *Ansätze* that only contain the lowest harmonics of the z dependence of the director. Those *Ansätze* allow a correct calculation of the position of the second-order transition line from the uniform to the homogeneous distorted state, but the actual coordinates of the multicritical point and of the coexistence and second-order transition lines that border the periodic distorted state may be off by as much as 5%. A test was made with the inclusion of the second harmonic in the analyses of the pure splay geometry increasing the value of r_c from 0.297 95... obtained with the first harmonic to 0.298 72...; the exact value is 0.303 25... [3].

In conclusion, we found that the multicritical point in the phase diagram associated with the Fréedericksz transitions in the splay-twist mixed geometry changes its nature for a value of r around 0.264 77... . This change at r_T is equivalent to a tricritical behavior when considering an analogy with the classical tricritical model with the wave vector playing the role of an order parameter and the magnetic field squared the role of a potential.

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